<u>The wrong way of proving AP /GP…</u>

In order to prove a series forms an AP we often use the following 2 steps:

1) $T_n = S_n - S_{n-1}$
2) Show that $T_n - T_{n-1}$ is a constant, then T_n forms an AP.

This is a wrong prove. Let's just use a counter-example.

If $S_n = n^2+1$, using $T_n = S_n - S_{n-1}$, we shall obtain

$$T_n = 2n-1 \text{ and } T_n - T_{n-1}=2 \text{ i.e. common difference =2=constant.}$$

But using $T_1 = S_1$ and $T_n = S_n - S_{n-1}$ we shall obtain $T_1=2$, $T_2=3$, $T_3=5$,… clearly this does not form an AP.

So what went wrong?

When we use $T_1 = S_1$ and $T_n = S_n - S_{n-1}$, when n=1 it means $T_1 = S_1 - S_0$, and if $T_1 = S_1$ it then implies that $S_0=0$. Hence $S_0=0$ is another condition that we need to satisfy before we can conclude that it forms an AP.

In conclusion, in proving that a series forms an AP we need the following 3 steps instead:

1) $T_n = S_n - S_{n-1}$
2) Show that $T_n - T_{n-1}$ is a constant, then T_n forms an AP.
3) $S_0=0$.

<u>What about GP?</u>

Using similar argument, we can arrive at the following conclusions:

In proving that a series forms a GP we need the following 3 steps instead:

1) $T_n = S_n - S_{n-1}$
2) Show that $\frac{T_n}{T_{n-1}}$ is a constant, then T_n forms a GP.
3) $S_0=0$.

Actually there is a simple way to argue why $S_0=0$? It is similar to why $T_1 = S_1$?

S_n refers to sum of first n terms, so S_0 refers to sum of zero term i.e. nothing to sum. Hence $S_0=0$.

Are there any other ways to prove AP/GP?

We can also try to fit the given S_n formula to fit AP/GP S_n formulas (and identify 1st terms, common difference/ratio) since these 2 S_n formulas satisfy $S_0=0$.

Mr GOH